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Analytical Estimation of Input RMS current ripple and Input filter design of Matrix Converter

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***Abstract*—A filter is required to remove the high frequency switching ripple present in the input current of the matrix converter. A design procedure for a single stage L-C filter is presented in this paper based on the specifications of allowable ripple in the grid current and distortion in the input voltage of the matrix converter. This requires an accurate estimation of the ripple component present in input line current. An analytical method to estimate this switching ripple is presented, and the converter is modeled for fundamental frequency. A damping resistor is designed to damp the oscillations due to LC resonance, with minimum power loss. The converter is simulated in MATLAB/Simulink and the filter design results are verified.**

***Index Terms*—Input RMS ripple, Indirect matrix converter, total harmonic distortion, input power factor**

I. INTRODUCTION

The matrix converter converts balanced three phase ac to three phase PWM ac of desired magnitude and frequency. Its unique properties like bidirectional power flow, single stage power conversion, open loop input power factor correction and most importantly minimum energy storage requirements which eliminates the bulky dc-link capacitor, give it an upper hand in applications like adjustable speed drives. However the switching of the converter injects high frequency current harmonics into the grid. Hence a passive LC filter is required to eliminate these higher order harmonics. An accurate estimation of the input current ripple is important for the filter design.

Analytical expression for the instantaneous input current based on modulation theory is given in [1], [2]. Computation of RMS ripple using these methods will require calculations with complicated Bessel functions. [3] provides a design for multistage L-C filter based on cost function optimization using genetic algorithms with different constraints like maximum amplitude of input ripple, minimum power factor, maximum losses in damping resistor, etc. [4], [5] derive ripple estimation from simulation to design the input filter. In [6], the switching ripple is approximated to be equal in magnitude to the fundamental component, and [7] designs the filter based on fundamental components. The need and design of damping resistor is explained in [8]. [9] analytically estimates the input ripple for a simpler situation of a current source inverter.

This paper presents an analytical approach to estimate the input RMS ripple current from the modulation strategy as a function of output power factor and modulation index, and is independent of input and output frequencies. Section I describes the modulation strategy and modelling of matrix converter for filter design. In section II the filter design is explained in detail, followed by Simulation and Conclusion in Section III and IV respectively.

II. ANALYSIS

A. Indirect Modulation of Matrix Converter

The section briefly describes the modulation technique for which the RMS input current ripple has been computed. This technique is based on the indirect modulation of matrix converter as explained in [10], [11]. In indirect modulation, a matrix converter is replaced by two fictitious converters as shown in Fig. 1. The grid side converter acting as a current source inverter(CSI), and a load side converter acting as a voltage source inverter(VSI) are connected through a virtual dc link. The front end converter rectifies input 3-phase utility to virtual dc, from which the output converter generates balanced 3-phase voltage of desired magnitude and frequency.

The space vectors of the CSI stage consists of six active current space vectors as shown in Fig. 3 where [a b] refers to the switching state when switch S_{aP} and S_{bN} are on. Similarly Fig. 4 shows the six active voltage space vectors of the VSI where [1 0 0] refers to a switching state when the switches S_{AP}, S_{BN} and S_{CN} are on.

In one sampling cycle, the input reference current vector is generated from two adjacent active vectors and one zero vector, whose duty ratios are given by (1), where m_I is the ratio of peak of the fundamental component of the input current to the virtual dc link current. Similarly, the output reference voltage space vector is generated by applying two active vectors and a zero vector and their duty ratios are given by (2), where m_V is

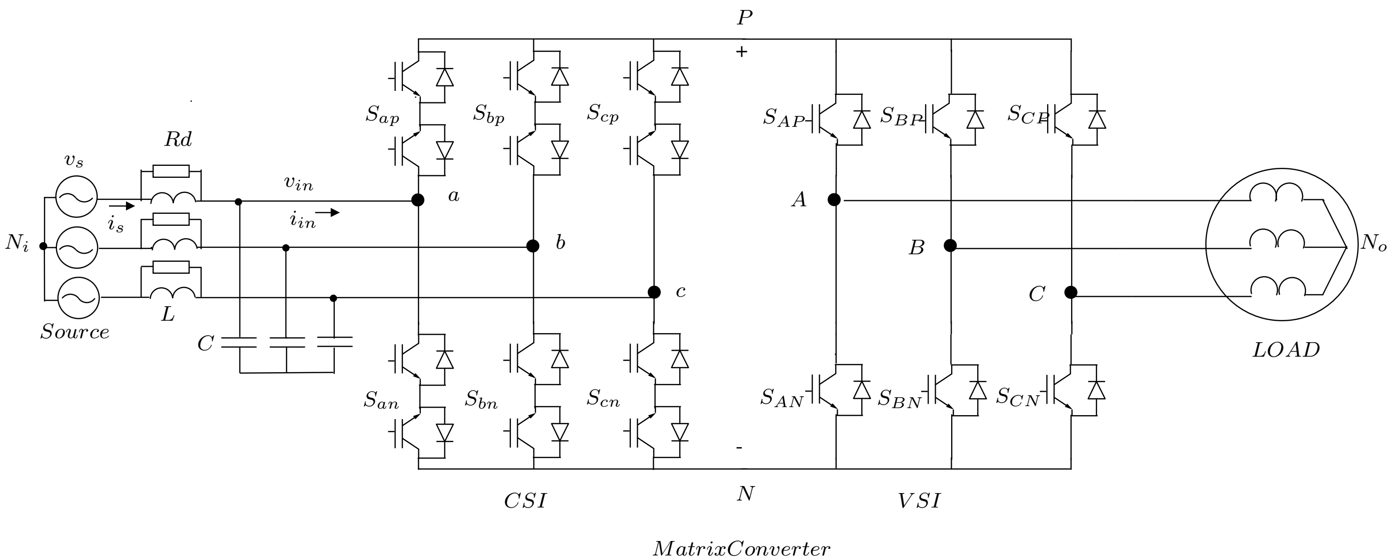

Fig. 1: Circuit diagram of the power electronic converter system

the ratio of peak of the fundamental component of the output voltage to the dc link voltage. The switching sequence applied over a cycle is given in Fig. 2.

$$\begin{aligned} dI_1 &= m_I \sin\left(\frac{\pi}{3} - \beta\right) \\ dI_2 &= m_I \sin\beta \end{aligned} \tag{1}$$

$$\begin{aligned} dV_1 &= \sqrt{3} m_V \sin\left(\frac{\pi}{3} - \alpha\right) \\ dV_2 &= \sqrt{3} m_V \sin\alpha \end{aligned} \tag{2}$$

CSI	I_1			I_2			0
VSI	V_1	V_2	0	V_1	V_2	0	0
Dutycycle	$dI_1 dV_1$	$dI_1 dV_2$	0	$dI_2 dV_1$	$dI_2 dV_2$	0	0

0 $\quad \frac{t}{T_s} \longrightarrow$ 1

Fig. 2: Switching sequence of Matrix converter

B. Modelling of Matrix Converter for Filter design

For the input filter design, it is essential to model the switching converter for different frequency components. For the input filter, the matrix converter appears to be a voltage dependent current source. In this section, the matrix converter is modelled as a current source both for the fundamental and the higher harmonic components (multiples of switching frequency).

The average input current vector is aligned along the input voltage vector in order to get unity power factor. So for the fundamental component of the input frequency, the matrix converter can be modelled as a resistive load, R_e. The peak of the average output load voltage V_o can be written in terms of the peak of the input line to neutral voltage V_{in} as in (3). Similarly the peak of average input current I_{in} can be written in terms of the peak of output load current I_o as in (4) where ϕ_o is the output power factor angle. Using these two equations, the effective resistance of the matrix converter can be expressed in terms of the output load impedance ($|Z_L| = \frac{V_o}{I_o}$) as shown in (5).

$$V_o = \frac{3}{2} m_I m_V V_{in} \tag{3}$$

$$I_{in} = \frac{3}{2} m_I m_V I_o \cos\phi_o \tag{4}$$

$$R_e = \frac{V_{in}}{I_{in}} = \frac{Z_L}{(\frac{3}{2})^2 (m_I m_V)^2 \cos\phi_o} \tag{5}$$

In order to model the matrix converter for higher order harmonics or for the ripple component of input current, the total RMS of the input current is analytically computed. As a first step, it is assumed that the output frequency is same as the input frequency. In order to simplify the computation, it is also assumed that the average output voltage vector was aligned along vector V_1 when the average input current vector was aligned along vector I_1. Through simulation and calculations, it is confirmed that the total RMS current computed with the previous assumptions remains unchanged even when the vectors are not aligned or the output and input frequencies are different, as can be seen in Fig. 6.

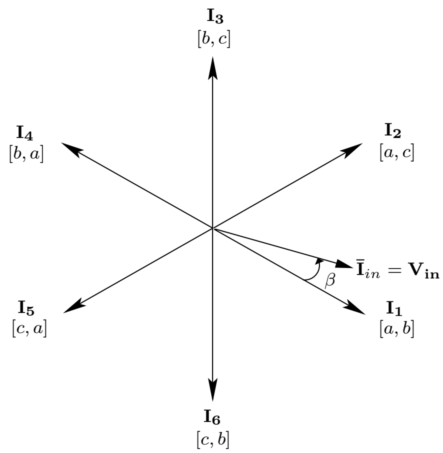

Fig. 3: Current space vectors produced by CSI

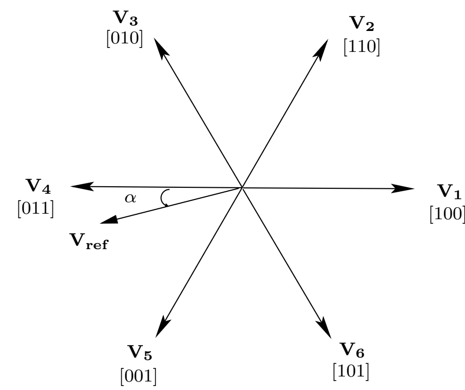

Fig. 4: Voltage space vectors produced by VSI

When the average input current space vector is in the first sector, over one sampling cycle the instantaneous input current i_a is composed of output line currents i_A and $-i_C$. As shown in Fig. 2 one sampling cycle is composed of time periods $dI_1dV_1T_s$, $dI_1dV_2T_s$, $dI_2dV_1T_s$ and $dI_2dV_2T_s$. During time period $dI_1dV_1T_s$, vector V_1 is applied to the output converter. So the virtual dc link current is same as the output phase A current. The switching state of the input converter during the same period is [a b]. So the input phase a is connected to the virtual dc link. This implies during this time period, i_a is equal to i_A. Similarly, in $dI_1dV_2T_s$ period $i_a = -i_C$, in $dI_2dV_1T_s$ period $i_a = i_A$ and in $dI_2dV_2T_s$ period $i_a = -i_C$ respectively. Here we assume the output frequency is much smaller compared to the switching or the sampling frequency of the converter, such that the output currents over a sampling cycle remains constant. So the square RMS of the input line current i_a in the first sector is given by (6). Similarly it is possible to obtain the expressions for the square RMS of the input line current i_a over a sampling cycle when the input current vector is in second sector (7) and third sector (8) respectively. These expressions repeat for other three sectors. Assuming the output line currents are balanced sinusoidal and lagging the output voltage vector by a power factor angle of ϕ_o, and using (1) and (2), it is possible to obtain the RMS of the input line current i_a over one sector, (9). Finally the actual total RMS over one fundamental cycle of i_a can be obtained from the RMS of each sector using (10). Hence the input RMS current is obtained as a function of load power factor and modulation index of the matrix converter as shown in (11). The RMS of switching ripple current is obtained by subtracting the fundamental current from total input RMS current (12). This expression for RMS current ripple remains unchanged with variation in output frequency and the alignment angle. This is confirmed through simulation (6) and calculations. Fig. 5 shows the variation of input RMS current with change in modulation index and load power factor.

$$\begin{aligned} i^2_{a_{RMS,T_s}} &= (dV_1dI_1 + dV_1dI_2)(i_A)^2 \\ &+ (dV_2dI_1 + dV_2dI_2)(-i_C)^2 \end{aligned} \tag{6}$$

$$i^2_{a_{RMS,T_s}} = dV_1dI_1(-i_C)^2 + dV_2dI_1(i_B)^2 \tag{7}$$

$$i^2_{a_{RMS,T_s}} = dV_1dI_2(i_B)^2 + dV_2dI_2(i_A)^2 \tag{8}$$

$$I^2_{aRMS_{SECTOR}} = \frac{1}{\pi/3}\int_{SECTOR} i^2_{aRMS,Ts}d(\omega_o t) \tag{9}$$

$$I^2_{aRMS} = \frac{1}{3}\sum_{i=1,2,3} I^2_{aRMS_{SECTORi}} \tag{10}$$

$$\begin{aligned} I^2_{a_{RMS}} &= \frac{3\sqrt{3}m_Im_VI_o^2}{\pi^2}\left(\frac{\pi\sqrt{3}}{12}+\frac{3}{8}\right)(1+\cos 2\phi_o) \\ &+ \frac{3\sqrt{3}m_Im_VI_o^2}{\pi^2}\left(\frac{\pi}{12}-\frac{\sqrt{3}}{16}\right)\sin 2\phi_o \end{aligned} \tag{11}$$

$$I^2_{a_{SWRMS}} = I^2_{a_{RMS}} - \left(\frac{3}{2}m_Im_VI_o\cos\phi_o\right)^2 \tag{12}$$

III. FILTER DESIGN

A passive L-C filter is designed to provide attenuation of the ripple current. In order to avoid resonance occuring due to L-C section, a damping resistor is introduced in parallel with L. The matrix converter is modelled as an equivalent resistance R_e for the fundamental frequency Fig. 7. For the ripple component, the matrix converter is modelled as a sinusoidal current source at the sampling frequency $1/T_s$ as shown in Fig. 8. The

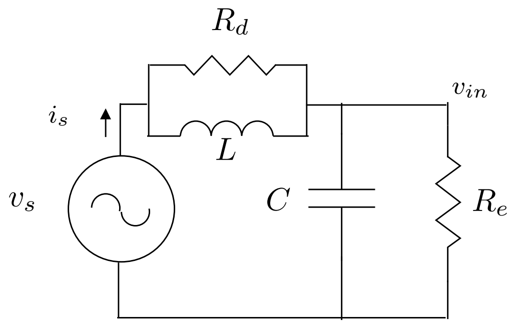

Fig. 7: Per-phase equivalent circuit at fundamental frequency

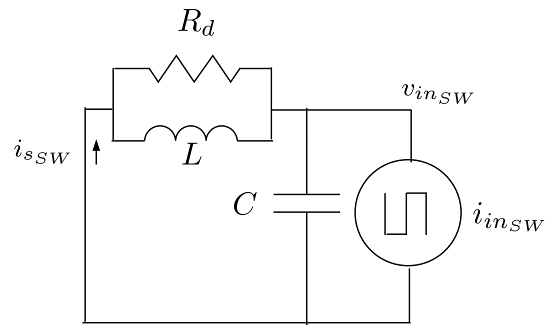

Fig. 8: Per-phase equivalent circuit at switching frequency

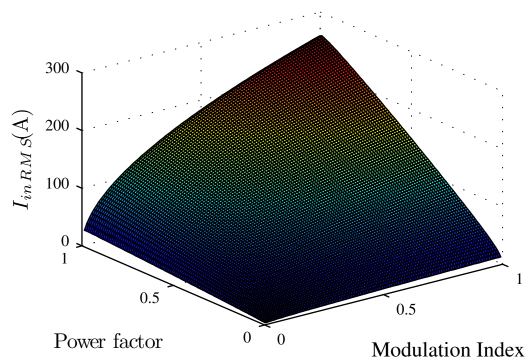

Fig. 5: Simulation results: Input RMS current

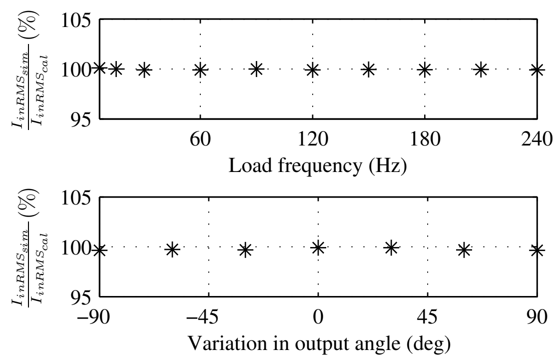

Fig. 6: Simulation result: Input RMS current with variation of output frequency and alignment angle

RMS of this current source is assumed to be given by (12). Due to pulse width modulation the first group of dominant harmonic components in the input current waveform appears to be at and around the sampling frequency. The other harmonic components with appreciable magnitude occurs at and around multiples of sampling frequency. Due to this, from the filter design perspective we may assume that the entire energy in the input current other than the fundamental is concentrated at the switching frequency. The L-C filter is designed such that the RMS of the ripple component of the source current, $I_{s_{SWRMS}}$ must be within a limit in order to maintain a particular THD. According to IEEE 519, THD in the source current must be less than 5%. For the proper operation of the matrix converter, the input voltage of the matrix converter must have a limited amount of higher harmonic ripple, $V_{in_{SWRMS}}$. Analyzing circuit as shown in Fig. 8 it is possible to express both of these ripple components in terms of $I_{in_{SWRMS}}$. Analyzing the circuit, for the fundamental component it is possible to compute the ratio of the power loss in the damping resistor to the total output power of the converter (15). For a given THD in the source current, RMS ripple of the input voltage waveform of the matrix converter and percentage loss in the damping resistor, (13), (14) and (15) can be solved simultaneously to determine L, C and R_d. It is also necessary that the source current must be drawn close to unity power factor or the angle θ_i must be close to zero (16) and the voltage drop in the fundamental component across the filter must be negligibly small or the ratio of fundamental component of source voltage to that of the input voltage of the matrix converter must be close to unity (17).

$$V_{in_{SWRMS}} = \frac{I_{in_{SWRMS}}}{\sqrt{(\omega_s C - \frac{1}{\omega_s L})^2 + \frac{1}{R_d^2}}} \tag{13}$$

$$I_{s_{SWRMS}} = \frac{I_{in_{SWRMS}}}{\sqrt{1 + \frac{(1-\omega_s^2 LC)^2 - 1}{\left(1 + \frac{\omega_s^2 L^2}{R_d^2}\right)}}} \tag{14}$$

$$\frac{P_{loss}}{P} = \frac{1}{3V_{in}} \left(\frac{\omega L}{\sqrt{\omega^2 L^2 + R_d^2}} \right) \tag{15}$$

$$\theta_i = \tan^{-1} \omega C R_e + \tan^{-1} \frac{\omega L}{R_d} - \tan^{-1} \frac{\omega L (R_e + R_d)}{R_e R_d (1 - \omega^2 LC)} \tag{16}$$

$$\frac{V_{in}}{V_s} = \frac{R_e \sqrt{R_d^2 + \omega^2 L^2}}{\sqrt{\omega^2 L^2 (R_e + R_d)^2 + R_e^2 R_d^2 (1 - \omega^2 LC)^2}} \tag{17}$$

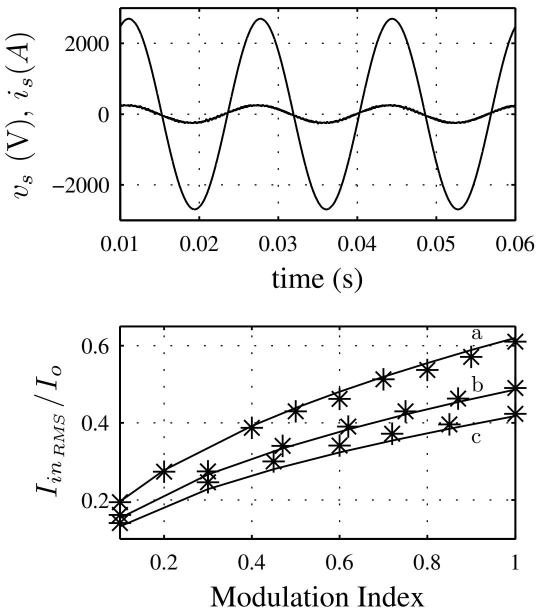

Fig. 9: Simulation results: a) Filtered source voltage and current b) Variation of input RMS current with modulation index and output power factor ($\cos\phi_o$ of a: 0.8, b: 0.6, c: 0.5)

IV. SIMULATION

The power electronic converter system as shown in Fig. 1 with the modulation strategy described in Section II is simulated in MATLAB/Simulink with a set of parameters as shown in Table I. The input filter components L, C and R_d are designed according to Section III. Fig. 12 shows the output line to neutral voltage and line current which is almost sinusoidal. Fig. 11 shows the input voltage and input current of the matrix converter. Frequency spectrum of various voltage and current waveforms are plotted in Fig. 10. From these two figures, it is evident that the voltage drop across the filter is negligibly small and the input voltage to the matrix converter is almost sinusoidal. Fig. 9(a) shows the source voltage and line current. It can be seen that the line current is almost in phase with the voltage and nearly sinusoidal. The frequency spectrum of the input source current confirms this observation. Fig. 9(b) provides $I_{in_{RMS}}/I_o$ as a function of modulation index and power factor. The simulated points confirm the analytically predicted continuous plots. This verifies the analytical estimation of the RMS input current described in Section II. Table II shows a comparison between different variables obtained from analysis and simulation.

V. CONCLUSION

A design procedure to determine the input L-C filter for a matrix converter is presented. A novel strategy to analytically estimate the input RMS ripple current for the converter is proposed. This computation is independent of the ratio of the input to the output frequency, and depends only on the modulation index and output power factor. The designed filter provides sufficient THD elimination and ensures near unity power factor, negligible drop across filter and minimum loss across damping resistor. The matrix converter is simulated and results are verified.

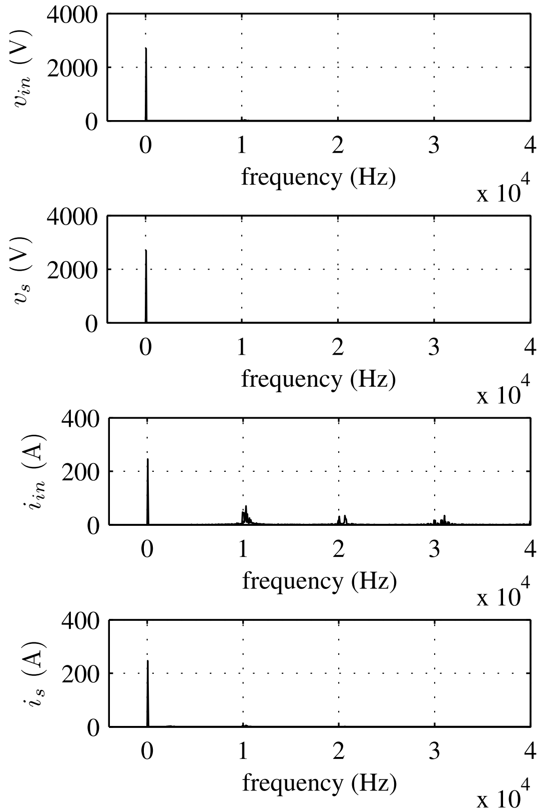

Fig. 10: Simulation results: Frequency spectrum

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TABLE I: Parameters

$V_{s_{LL-RMS}}$	$3.3kV$
f_i	$60Hz$
P_o	$1MW$
$\cos\phi_o$	0.8
m_I	1
m_V	$\frac{1}{\sqrt{3}}$
$f_s = \frac{1}{T_s}$	10kHz
f_o	$30Hz$
L	0.175 mH
C	37.32 μF
R_d	10 Ω
R_e	10.89 Ω

TABLE II: Simulation Results : A Comparison

VARIABLES	ANALYTICAL	SIMULATED
V_o	2323.5V	2322.8V
I_o	357.12A	357.00A
I_{in}	247.4A	249.5A
$I_{in_{RMS}}$	216.38A	216.20A
$I_{in_{SWRMS}}$	127.33A	126.20A
V_{in}	2694.4V	2695.0V
$V_{in_{SWRMS}}$	53.8V	44.7V
$I_{s_{SWRMS}}$	5.0A	7.2A
$\cos\phi_i$	0.9973	0.9971
$\frac{V_{in}}{V_s}$	1.0002	1.0000
$P_{loss}(\%)$	0.000283	0

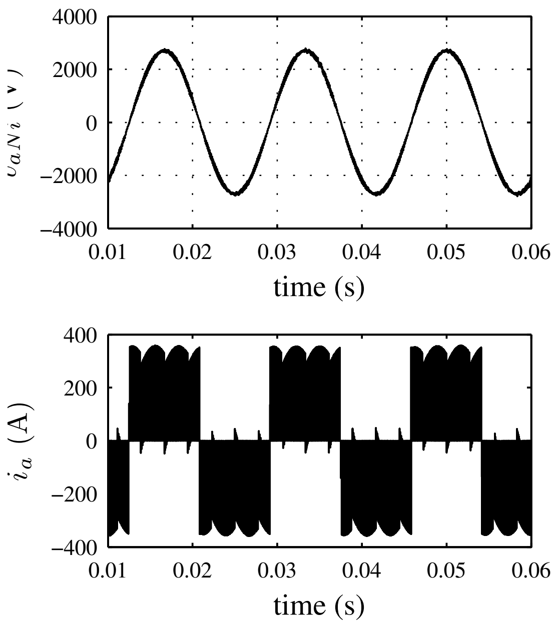

Fig. 11: Simulation results: a) Input line to neutral voltage b) Input line current

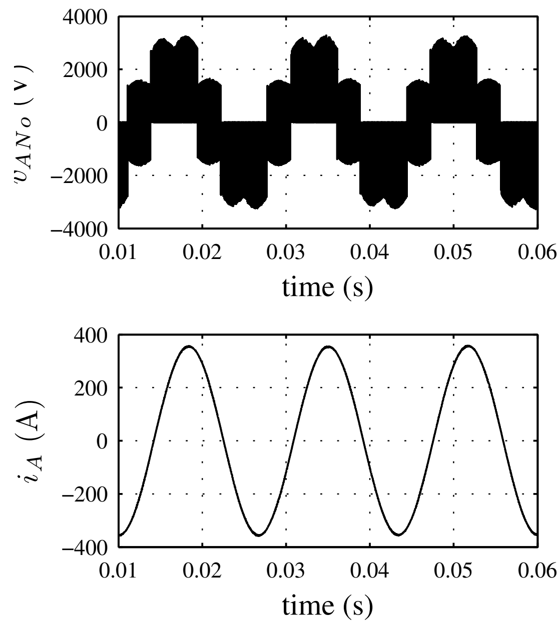

Fig. 12: Simulation results: a) Output line to neutral voltage b) Output line current

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